

A TECHNIQUE FOR PREDICTING THE THERMAL CONDUCTIVITY OF SUSPENSIONS, EMULSIONS AND POROUS MATERIALS

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Abstract—A theoretical technique to predict the thermal conductivity of heterogeneous mixtures, previously derived by the writers, has been applied to suspensions, emulsions and porous materials. Based on the data available from the literature, the technique is shown to predict the values of thermal conductivity within 18.2 per cent for suspensions, 9.5 per cent for emulsions and 9.9 per cent for porous materials.

NOMENCLATURE

- B , constant in the parabolic distribution ;
 C , constant in the parabolic distribution ;
 d , diameter of pore [cm] ;
 e , emissivity ;
 k , thermal conductivity [Btu/h ft °F or cal/s cm °C] ;
 P , phase volume fraction ;
 P_{cs} , cross-sectional pore fraction ;
 P_p , longitudinal pore fraction ;
 Q , ratio of k_s/k_a ;
 R , thermal resistance [h °F/Btu] ;
 T , absolute temperature [°R or °K].

Greek letters

- γ , geometrical pore factor ;
 σ' , radiation constant [erg/cm² s °C⁴].

Subscripts

- a , air ;
 c , continuous phase ;
 d , discontinuous phase ;
 e , equivalent ;
 m , mean ;
 s , solid.

INTRODUCTION

A THEORETICAL technique to predict the thermal conductivity of heterogeneous solid mixtures has been developed and reported by the writers [1]. The technique considers the aggregate as a single model as opposed to conventional methods which consider the aggregate as a repetitive array of unit cells. The key assumption of the technique is that the discontinuous phase can be rearranged in the continuous phase, such that the former can be described by a parabolic distribution function with appropriate boundary conditions.

The technique was developed assuming conduction as the predominant mode of heat transfer. The purpose of this paper is to show the applicability of the technique to suspensions, emulsions and porous materials. The assumption that radiative and convective heat transfer are negligible is maintained.

A number of studies [2-13] has been conducted to develop techniques for predicting the thermal conductivity of heterogeneous systems. Table 1 lists some of the equations resulting from the above studies. These equations will be examined and used in conjunction

Table 1. Equations for predicting conductivity of mixtures

A. Maxwell [2]—two-phase mixtures

$$k_m = \frac{k_c[k_d + 2k_c - 2P_d(k_c - k_d)]}{k_d + 2k_c + P_d(k_c - k_d)}. \quad (4)$$

B. Jefferson [5]—two-phase mixtures

$$k_m = k_c \left[1 - \frac{\pi}{4(1 + 2n)^2} \right] + \frac{\pi}{4(1 + 2n)^2} \left[\frac{(0.5 + n)k_d k_c}{0.5k_c + nk_d} \right]. \quad (5)$$

where

$$k_d = k_c k_a \left[\frac{2k_d}{(k_d - k_c)} \ln \frac{k_d}{k_c} - \frac{2}{k_d - k_c} \right]$$

and

$$n = 0.403P_d^{-1/2} - 0.5.$$

C. Russell [10]—two-phase mixtures (porous materials)

$$k_m = k_c \left[\frac{P_d^\dagger + \frac{k_c}{k_d}(1 - P_d^\dagger)}{P_d^\dagger - P_d + \frac{k_c}{k_d}(P_d + 1 - P_d^\dagger)} \right]. \quad (6)$$

D. Loeb [11]—porous materials considering radiation

$$k_m = k_s \left[(1 - P_{cs}) + \frac{P_{cs}}{P_j k_s / 4\sigma' \epsilon \gamma d T_m^3 + (1 - P_j)} \right]. \quad (7)$$

E. Francl and Kingery [12]—porous materials

$$k_m = k_s(1 - P_d). \quad (8)$$

F. Eucken [13]—porous materials

$$k_m = k_s \frac{1 + 2P_d \frac{1 - Q}{2Q + 1}}{1 - P_d \frac{1 - Q}{2Q + 1}}, \quad (9)$$

where

$$Q = \frac{k_s}{k_a}.$$

G. The Writers' equations [1] can be expressed as

A. $k_c > k_d$.

$$R_e = \frac{2}{\sqrt{\{C(k_d - k_c)[k_c + B(k_d - k_c)]\}}} \tan^{-1} \frac{B}{2\sqrt{\{k_c + B(k_d - k_c)\}}} + \frac{1 - B}{k_c}, \quad (10)$$

where

$$B = \sqrt{(3P_d/2)}, \quad C = -4\sqrt{[2/(3P_d)]}$$

and

$$R_e = 1/k_m.$$

B. $k_d > k_c$.

$$R_e = \frac{1}{\sqrt{\{C(k_c - k_d)[k_c + B(k_d - k_c)]\}}} \ln \frac{\sqrt{[k_c + B(k_d - k_c)]} + \frac{B}{2}\sqrt{[C(k_c - k_d)]}}{\sqrt{[k_c + B(k_d - k_c)]} - \frac{B}{2}\sqrt{[C(k_c - k_d)]}} + \frac{1 - B}{k_c}, \quad (11)$$

Table 1. Continued

where $R_e = \frac{1}{k_m}$. If $k_d \gg k_c$, then equation (11) takes a very simple form, i.e.

$$R_e = \frac{1 - B}{k_c} \quad (12)$$

as shown in [1].

with data from the literature. Hamilton [6] has classified mixtures as follows:

Class I: $k_c \gg k_d$ or $k_c/k_d > 100$

Class II: $k_d \gg k_c$ or $k_d/k_c > 100$

Class III: $1 < k_c/k_d < 100$

Class IV: $1 < k_d/k_c < 100$

and these classifications will be used in this paper.

Suspensions, emulsions and porous materials are discussed as follows.

SUSPENSIONS

A suspension refers to solid particles suspended in a liquid. The density of the discontinuous phase must be comparable to that of the continuous phase for a suspension to exist. A number of techniques have been proposed to predict the thermal conductivity of suspensions. Maxwell [2], Rayleigh [3], Bruggeman [4], Jefferson [5], Hamilton [6], Tsao [7], Baxley [8] and Russell [10] constitute some of the major contributors. Table 2 presents some existing experimentally determined values of conductivity for various suspensions and predicted values of thermal conductivity as determined from the techniques developed by Maxwell [2], Baxley [8], Russell [10] and equations (11) and (12) of Ref. [1]. All the predicted values by Maxwell, Baxley and Russell in Table 2 are taken directly from Table 15 of Ref. [8]. The average percentage of deviation of predicted values from the measured values of thermal conductivity is

shown in Table 5 (A). None of these methods predict a consistently satisfactory result. In general, as the ratio of k_d/k_c increases, the percentage of deviation from the measured value of thermal conductivity increases. The ratio of k_d/k_c in Class IV mixtures is less than that of Class II mixtures. Therefore, the existing equations including the writer's equations predict the thermal conductivity of Class IV mixtures better than the Class II mixtures.

Many suspensions are Class II mixtures, i.e. $k_d \gg k_c$. Hence equation (12) which is an approximation of equation (11) takes a very simple form for Class II mixtures and can be used to predict the thermal conductivity as satisfactorily as any of the other techniques. As can be seen equation (12) predicts almost the same values as equation (11). Equation (12) is not a function of k_d as can be seen. Thus the thermal conductivity of the discontinuous phases is insignificant in comparison to the conductivity of the mixture in Class II mixtures. Equations (11) and (12) give acceptable results, and equation (12) is simpler in form than any of the other techniques.

EMULSIONS

An emulsion refers to liquid droplets distributed throughout an immiscible liquid. The characteristics of emulsions [9] are;

1. The density of the continuous phase and discontinuous phase should be comparable.
2. The discontinuous phase droplets are spherical.
3. The continuous phase is viscous enough

Table 2. Comparison of the predicted thermal conductivities with experimental data of suspensions

| Classification of mixtures | System of mixtures | | P_d | k_{exp}^* | Reference of experiment | Maxwell | | |
|---------------------------------|------------------------|--------------------------------|------------------------|---------------|-------------------------|---------|-------------------------|--------|
| | Solid | Liquid | | | | k_m | Percentage of deviation | |
| Class I mixtures | Graphite $k_d = 93$ | Water $k_c = 0.325$ | 0.161 | 0.685 | | 0.5089 | -25.60 | |
| | | | 0.247 | 0.978 | | 0.6404 | -34.50 | |
| | | | 0.321 | 1.237 | | 0.7789 | -37.00 | |
| | Graphite $k_d = 93$ | Silicone oil $k_c = 0.0875$ | 0.194 | 0.241 | Jefferson [5] | 0.1505 | -37.60 | |
| | | | 0.251 | 0.282 | | 0.1751 | -37.90 | |
| | | | 0.310 | 0.344 | | 0.2050 | -40.40 | |
| | | | 0.375 | 0.453 | | 0.2443 | -46.10 | |
| | | | 0.423 | 0.546 | | 0.2790 | -28.90 | |
| | Class II mixtures | Aluminum $k_d = 118$ | Water $k_c = 0.380$ | 0.055 | 0.440 | | 0.4457 | 1.29 |
| | | | | 0.115 | 0.560 | | 0.5365 | -5.98 |
| | | | | 0.175 | 0.810 | | 0.6190 | -23.60 |
| | | | | 0.210 | 1.000 | | 0.6794 | -32.10 |
| Copper $k_d = 221$ | | Water $k_c = 0.380$ | 0.055 | 0.430 | Johnson [14] | 0.4460 | 3.72 | |
| | | | 0.115 | 0.540 | | 0.5273 | -2.36 | |
| | | | 0.175 | 0.660 | | 0.6203 | -6.01 | |
| | | | 0.210 | 0.740 | | 0.6811 | -7.96 | |
| | | | 0.245 | 0.830 | | 0.7474 | -9.95 | |
| | | | 0.295 | 1.050 | | 0.8536 | -18.70 | |
| Iron $k_d = 37.8$ | | Lard $k_c = 0.114$ | 0.108 | 0.115 | | 0.1550 | 0.01 | |
| | | | 0.194 | 0.218 | | 0.1954 | -10.40 | |
| | 0.322 | | 0.339 | 0.2743 | | -19.10 | | |
| Zinc sulphate $k_d = 0.354$ | Lard $k_c = 0.114$ | 0.238 | 0.143 | Lees [15] | 0.1512 | 5.75 | | |
| | | 0.375 | 0.179 | | 0.1766 | -1.36 | | |
| | | 0.555 | 0.203 | | 0.2155 | 6.16 | | |
| Class IV mixtures | Marble $k_d = 1.72$ | Vaseline $k_c = 0.107$ | 0.250 | 0.153 | | 0.1916 | 25.20 | |
| | | | 0.430 | 0.262 | | 0.2865 | 9.35 | |
| | | | 0.600 | 0.432 | | 0.4285 | -0.80 | |
| Selenium $k_d = 3$ | PPG $k_c = 0.081$ | 0.100 | 0.104 | Baxley [6] | 0.1057 | 1.45 | | |
| | | 0.200 | 0.123 | | 0.1360 | 7.53 | | |
| | | 0.300 | 0.182 | | 0.1741 | -4.51 | | |
| | | 0.400 | 0.244 | | 0.2233 | -8.57 | | |
| | | 0.500 | 0.281 | | 0.2893 | 3.07 | | |
| Average percentage of deviation | | | | | | 16.3 | | |

* Units of k : Btu/h ft²F.

| Baxley | | Russell | | Equation (11) | | Equation (12) | |
|--------|-------------------------|---------|-------------------------|---------------|-------------------------|---------------|-------------------------|
| k_m | Percentage of deviation | k_m | Percentage of deviation | k_m | Percentage of deviation | k_m | Percentage of deviation |
| 0.4646 | -32.20 | 0.7029 | 2.62 | 0.6250 | -8.75 | 0.638 | -6.86 |
| 0.6186 | -36.70 | 0.8595 | -12.10 | 0.8070 | -17.50 | 0.829 | -15.24 |
| 0.8762 | -29.20 | 1.0144 | -18.00 | 1.0370 | -16.20 | 1.065 | -13.90 |
| 0.1333 | -44.50 | 0.2070 | -14.10 | 0.1890 | -27.50 | 0.190 | -21.20 |
| 0.1722 | -38.90 | 0.2360 | -16.30 | 0.2240 | -20.50 | 0.226 | -19.88 |
| 0.2416 | -29.80 | 0.2696 | -21.60 | 0.2730 | -20.70 | 0.276 | -19.80 |
| 0.3792 | -16.30 | 0.3123 | -31.10 | 0.3450 | -23.70 | 0.350 | -22.75 |
| 0.7743 | 41.80 | 0.3492 | -36.00 | 0.4260 | -21.90 | 0.433 | -20.70 |
| 0.4179 | -5.01 | 0.6051 | 37.50 | 0.5260 | 19.55 | 0.530 | 21.00 |
| 0.4641 | -17.10 | 0.7304 | 30.40 | 0.6390 | 14.10 | 0.650 | 16.10 |
| 0.5646 | -30.30 | 0.8513 | 5.10 | 0.7630 | -5.80 | 0.779 | -3.83 |
| 0.6029 | -39.70 | 0.9246 | -7.54 | 0.8450 | -15.50 | 0.863 | -13.70 |
| 0.4166 | -3.11 | 0.6088 | 41.60 | 0.5290 | 23.00 | 0.533 | 23.80 |
| 0.4738 | -12.30 | 0.7347 | 36.10 | 0.6440 | 19.30 | 0.650 | 20.40 |
| 0.5392 | -18.30 | 0.8564 | 29.80 | 0.7700 | 16.70 | 0.779 | 18.00 |
| 0.6697 | -9.50 | 0.9303 | 25.70 | 0.8530 | 15.30 | 0.864 | 16.76 |
| 0.7363 | -11.29 | 1.0.079 | 21.40 | 0.9480 | 14.20 | 0.962 | 15.90 |
| 1.0025 | -4.53 | 1.1280 | 7.43 | 1.1150 | 6.20 | 1.134 | 8.00 |
| 0.1394 | 10.10 | 0.2151 | 38.80 | 0.1880 | 21.30 | 0.191 | 22.90 |
| 0.1786 | -18.10 | 0.2674 | 22.70 | 0.2430 | 11.46 | 0.248 | 13.76 |
| 0.3180 | -6.20 | 0.3574 | 5.43 | 0.3620 | 6.78 | 0.374 | 10.32 |
| 0.1407 | -1.64 | 0.1577 | 10.30 | 0.1530 | 7.00 | | |
| 0.1578 | -11.90 | 0.1829 | 2.21 | 0.1780 | -0.56 | | |
| 0.1789 | -11.90 | 0.2203 | 8.51 | 0.2190 | 7.74 | | |
| 0.1689 | 10.40 | 0.2325 | 51.90 | 0.2130 | 39.20 | | |
| 0.2473 | -5.63 | 0.3303 | 26.10 | 0.3340 | 27.60 | | |
| 0.3557 | -17.70 | 0.4683 | 8.39 | 0.6020 | 39.50 | | |
| 0.0964 | -7.50 | 0.1376 | 32.00 | 0.1213 | 16.40 | | |
| 0.1202 | -5.01 | 0.1776 | 39.50 | 0.1580 | 24.90 | | |
| 0.1585 | -13.10 | 0.2192 | 20.20 | 0.2069 | 13.50 | | |
| 0.2162 | -11.50 | 0.2709 | 10.90 | 0.2805 | 14.90 | | |
| 0.2958 | 5.39 | 0.3378 | 20.30 | 0.4067 | 45.00 | | |
| | 17.4 | | 21.6 | | 18.2 | | |

Table 3. Comparison of the predicted thermal conductivities with experimental data of emulsions

| Systems of mixtures | k_f^* | k_c | P_d | Temp. (°F) | Maxwell | | | Jefferson | | | Russell | | | Equation (11) | | |
|---------------------------------|---------|-------|-------|------------|---------|-------------------------|--------|-------------------------|--------|-------------------------|---------|-------------------------|-------|-------------------------|-------|-------------------------|
| | | | | | k_m | Percentage of deviation | k_m | Percentage of deviation | k_m | Percentage of deviation | k_m | Percentage of deviation | k_m | Percentage of deviation | k_m | Percentage of deviation |
| Cellulose in EPO | 0.269 | 0.078 | 0.1 | 40.0 | 0.0806 | 9.68 | 0.0861 | 6.83 | 0.092 | 14.50 | 0.091 | 12.65 | | | | |
| | 0.318 | 0.096 | 0.1 | 100.0 | 0.1100 | -0.91 | 0.1070 | -2.73 | 0.114 | 3.64 | 0.111 | 0.91 | | | | |
| | 0.332 | 0.110 | 0.1 | 140.0 | 0.1210 | 2.48 | 0.1210 | 0.00 | 0.128 | 5.78 | 0.126 | 4.13 | | | | |
| | 0.269 | 0.078 | 0.3 | 40.0 | 0.1040 | 9.61 | 0.1100 | 5.77 | 0.120 | 15.40 | 0.116 | 11.54 | | | | |
| | 0.318 | 0.096 | 0.3 | 100.0 | 0.1360 | 2.94 | 0.1350 | -0.74 | 0.146 | 7.35 | 0.141 | 3.68 | | | | |
| 0.332 | 0.110 | 0.3 | 140.0 | 0.1480 | 4.73 | 0.1510 | 2.03 | 0.161 | 8.78 | 0.157 | 6.08 | | | | | |
| Cellulose in PPG | 0.269 | 0.064 | 0.1 | 40.0 | 0.0672 | 11.16 | 0.0723 | 7.59 | 0.079 | 17.86 | 0.077 | -13.84 | | | | |
| | 0.318 | 0.087 | 0.1 | 100.0 | 0.1050 | -4.76 | 0.0971 | -3.24 | 0.105 | 0.00 | 0.102 | -2.86 | | | | |
| | 0.332 | 0.089 | 0.1 | 140.0 | 0.1040 | -1.92 | 0.0992 | -4.61 | 0.107 | 2.88 | 0.105 | 0.96 | | | | |
| | 0.269 | 0.064 | 0.3 | 40.0 | 0.0897 | 10.80 | 0.0953 | 6.25 | 0.106 | 18.20 | 0.101 | 12.60 | | | | |
| | 0.318 | 0.087 | 0.3 | 100.0 | 0.1350 | -3.70 | 0.1250 | -7.40 | 0.137 | 1.48 | 0.132 | -2.12 | | | | |
| 0.332 | 0.089 | 0.3 | 140.0 | 0.1570 | -15.30 | 0.1280 | -18.50 | 0.141 | -10.20 | 0.136 | -13.40 | | | | | |
| Water in petroleum solvent | 0.349 | 0.105 | 0.2 | 75.3 | 0.1540 | -11.70 | 0.1310 | -14.93 | 0.142 | -7.80 | 0.137 | -11.04 | | | | |
| | 0.356 | 0.093 | 0.2 | 86.3 | 0.1270 | -3.15 | 0.1180 | -7.09 | 0.131 | 3.15 | 0.126 | -0.79 | | | | |
| | 0.342 | 0.118 | 0.4 | 64.0 | 0.2250 | -18.70 | 0.1780 | -20.90 | 0.189 | -16.00 | 0.184 | -18.20 | | | | |
| | 0.352 | 0.100 | 0.4 | 80.2 | 0.1800 | -7.22 | 0.1610 | -10.55 | 0.174 | -3.34 | 0.169 | -6.11 | | | | |
| | 0.341 | 0.119 | 0.6 | 62.4 | 0.2690 | -16.00 | 0.2190 | -18.60 | 0.229 | -14.90 | 0.230 | -14.50 | | | | |
| 0.349 | 0.106 | 0.6 | 74.6 | 0.2340 | -6.84 | 0.2110 | -9.83 | 0.222 | -5.13 | 0.223 | -4.70 | | | | | |
| Water in mineral oil | 0.340 | 0.095 | 0.2 | 60.5 | 0.1660 | -25.30 | 0.1200 | -27.70 | 0.131 | -21.10 | 0.123 | -24.10 | | | | |
| | 0.352 | 0.086 | 0.2 | 80.1 | 0.1350 | -14.10 | 0.1110 | -17.80 | 0.123 | -8.90 | 0.118 | -12.60 | | | | |
| | 0.344 | 0.094 | 0.4 | 62.0 | 0.1970 | -19.30 | 0.1540 | -21.80 | 0.166 | -15.74 | 0.162 | -17.80 | | | | |
| | 0.347 | 0.090 | 0.4 | 71.1 | 0.1840 | -15.80 | 0.1500 | -18.50 | 0.163 | -11.40 | 0.158 | -14.10 | | | | |
| | 0.353 | 0.086 | 0.4 | 81.4 | 0.1690 | -10.06 | 0.1460 | -13.60 | 0.160 | -5.33 | 0.155 | -8.29 | | | | |
| 0.342 | 0.094 | 0.6 | 63.0 | 0.2580 | -20.95 | 0.1970 | -23.65 | 0.209 | -19.00 | 0.211 | -18.20 | | | | | |
| 0.348 | 0.089 | 0.6 | 72.7 | 0.2270 | -11.45 | 0.1940 | -14.52 | 0.206 | -9.25 | 0.209 | -7.93 | | | | | |
| 0.353 | 0.086 | 0.6 | 81.3 | 0.2000 | -0.50 | 0.1920 | -4.00 | 0.205 | 2.50 | 0.209 | 4.50 | | | | | |
| Average percentage of deviation | | | | | 10.0 | | 11.1 | | 9.6 | | 9.5 | | | | | |

* Units of k : Btu/h ft² F.

Table 4. Comparison of the predicted thermal conductivities with experimental data of porous materials

| Characteristics of porous materials | P_d | K_{exp} | Russell | | | Loeb | | | Eucken | | | Equation (10) | | |
|--|-------|------------|------------|-------------------------|-------------|-------------------------|------------|-------------------------|-------------|-------------------------|------------|-------------------------|-------------|-------------------------|
| | | | k_m | Percentage of deviation | k_m | Percentage of deviation | k_m | Percentage of deviation | k_m | Percentage of deviation | k_m | Percentage of deviation | k_m | Percentage of deviation |
| Alumina, Al ₂ O ₃ , with isometric pores at 200°C $k_c^* = 0.051$ [12] | 0.10 | 0.0450 | 0.0445 | -1.110 | 0.0460 | 2.22 | 0.0435 | -3.33 | 0.0445 | -1.11 | 0.0435 | -3.33 | 0.0445 | -1.11 |
| | 0.20 | 0.0400 | 0.0390 | -2.500 | 0.0410 | 2.50 | 0.0375 | -6.25 | 0.0373 | -6.75 | 0.0375 | -6.25 | 0.0373 | -6.75 |
| | 0.30 | 0.0350 | 0.0335 | -4.280 | 0.0360 | 2.86 | 0.310 | -11.44 | 0.0300 | -14.30 | 0.310 | -11.44 | 0.0300 | -14.30 |
| Alumina, Al ₂ O ₃ , with cylindrical pores of diameter 0.146 cm at 200°C $k_c^* = 0.051$ [12] | 0.10 | 0.0452 | 0.0448 | -0.885 | 0.0460 | 1.77 | 0.0436 | -3.54 | 0.0445 | -1.55 | 0.0436 | -3.54 | 0.0445 | -1.55 |
| | 0.20 | 0.0400 | 0.0390 | -2.500 | 0.0410 | 2.50 | 0.0376 | -6.00 | 0.0373 | -6.75 | 0.0376 | -6.00 | 0.0373 | -6.75 |
| | 0.30 | 0.0350 | 0.0340 | -2.860 | 0.0360 | 2.86 | 0.0304 | -13.15 | 0.0300 | -14.30 | 0.0304 | -13.15 | 0.0300 | -14.30 |
| Iwaki sandstone with air saturated at 50°C $k_c^\dagger = 1.6$ [17] | 0.10 | 1.25 | 1.424 | 13.90 | 1.45 | 16.00 | 1.376 | 10.10 | 1.40 | 12.00 | 1.376 | 10.10 | 1.40 | 12.00 |
| | 0.20 | 0.96 | 1.232 | 28.30 | 1.29 | 34.40 | 1.185 | 23.44 | 1.17 | 21.88 | 1.185 | 23.44 | 1.17 | 21.88 |
| | 0.30 | 0.76 | 1.056 | 39.00 | 1.13 | 48.70 | 0.976 | 28.40 | 0.94 | 23.70 | 0.976 | 28.40 | 0.94 | 23.70 |
| Diaspore brick $k_c/k_d = 50$ [18] | 0.10 | 0.88 k_c | 0.89 k_c | 1.136 | 0.905 k_c | 2.84 | 0.87 k_c | -1.14 | 0.895 k_c | 1.71 | 0.87 k_c | -1.14 | 0.895 k_c | 1.71 |
| | 0.20 | 0.76 k_c | 0.77 k_c | 1.316 | 0.805 k_c | 5.92 | 0.74 k_c | -2.63 | 0.732 k_c | -3.68 | 0.74 k_c | -2.63 | 0.732 k_c | -3.68 |
| | 0.30 | 0.66 k_c | 0.66 k_c | 0.000 | 0.705 k_c | 6.82 | 0.61 k_c | -7.50 | 0.588 k_c | -10.90 | 0.61 k_c | -7.50 | 0.588 k_c | -10.90 |
| Average percentage of deviation | | | | 8.2 | | 10.8 | | 9.8 | | 9.9 | | 9.8 | | 9.9 |

* Units of k : cal/s cm °C.

† Units of k : kcal/h m °C.

to avoid settling and coalescence, thus preventing convection.

4. The thermal conductivities of continuous phase and discontinuous phase of emulsions are of the same order of magnitude.

5. The volume fraction of the discontinuous phase, P_d , should be low to prevent coalescence.

The above characteristics are compatible with the assumptions made by Maxwell and Jefferson in deriving equations (4) and (5) to predict the thermal conductivities of two-phase mixtures. One would thus expect these two equations to predict satisfactory results for emulsions. In fact, they do predict satisfactory results as shown in the following paragraph.

A number of other equations have also been proposed to predict the thermal conductivity of emulsions. Rayleigh [3], Bruggeman [4], Hamilton [6], Tsao [7] and Russell [10] have developed some of these relations. Table 3 presents some experimental data of Nahas and Couper [9] for various emulsions. Predicted values of conductivity by Maxwell [2],

Jefferson [5], Russell [10] and equation (11) are compared with the experimental data. All the predicted values of Maxwell, Jefferson and Russell in Table 3 are taken from the Tables 1, 2 and 12 and Figs. 14 and 15 of Ref. [9]. The average percentage of deviation of predicted values from the measured values of thermal conductivity is shown in Table 5 (B). Equation (11) is seen to give satisfactory results.

POROUS MATERIALS

The mechanism of heat transfer in porous materials in general is very complicated and involves conduction as well as convection and radiation. Convection and radiation which occur in the pore spaces can be neglected for small pore size and low or intermediate temperature. One of the important aspects of the thermal conductivity in porous materials as indicated by Barrett [16] is the effect of the orientation of the pores on the direction of heat flow. The porous materials studied in this investigation primarily belong to the "air-cell" type of mixture rather than "granular-solid" type of mixture. Contact resistance can be neglected for the "air-cell" type of porous materials. A number of equations have been proposed to predict the thermal conductivity of porous materials. Equations by Russell [10], Loeb [11], Francl and Kingery [12] and Eucken [13] are the most used. Table 4 presents the experimental data for porous materials by Francl and Kingery [12], Sugawara and Yoshizawa [17], and Austin [18] and the predicted values by Russell [10], Loeb [11], Eucken [13] and equation (10). All the predicted values by Russell, Loeb and Eucken of the first two mixtures in Table 4 are taken from the Figs. 5, 6, 11 and 12 of Ref. [12]. The average percentage of deviation of predicted values from the measured values of the thermal conductivity is shown in Table 5 (C). The Russell and Loeb techniques predict values within a range of 2-3 per cent of measured values for the first two mixtures in Table 4. These samples contain known distributions of the pores rather

Table 5. Summary of the average percentage of deviation of predicted values from the measured values of thermal conductivity

| Predicted values of thermal conductivity | Average percentage of deviation from the measured values of thermal conductivity |
|--|--|
| A. Suspensions as given in Table 2 | |
| Maxwell | 16.3 |
| Baxley | 17.4 |
| Equation (11) | 18.2 |
| Russell | 21.6 |
| B. Emulsions as given in Table 3 | |
| Equation (11) | 9.5 |
| Russell | 9.6 |
| Maxwell | 10.0 |
| Jefferson | 11.1 |
| C. Porous materials as given in Table 4 | |
| Russell | 8.2 |
| Eucken | 9.8 |
| Equation (10) | 9.9 |
| Loeb | 10.8 |

than a random distribution of the pores. The technique of [1] is for a random distribution as approximated by the assumed parabolic distribution. Francl and Kingery [12] point out that Russell's equation, equation (6), is not applicable to anisometric pores but Loeb's equation, equation (7), is applicable to both isometric and anisometric pores. Also, for low temperature Loeb's equation can be reduced to equation (8) simply by neglecting the radiation effect in the pores. Again, one sees that the proposed technique gives acceptable results.

SUMMARY

Equations (10)–(12) have been compared with existing equations for predicting the thermal conductivity of two-phase mixtures. These equations are seen to give acceptable results for suspensions, emulsions and porous materials.

Table 6. Summary of the average percentage of deviation of the predicted thermal conductivities by equations (10) and (11) with some of the existing methods for various systems of two-phase-mixtures

| Type of mixtures | Average percentage of deviation predicted by equations (10) and (11) | Average percentage of deviation predicted by the best method investigated in this study |
|----------------------|--|---|
| Solid-solid mixtures | 6.4 [1] | 7.5 by Maxwell, equation (4) |
| Suspensions | 18.2 | 16.3 by Maxwell, equation (4) |
| Emulsions | 9.5 | 10.0 by Maxwell, equation (4) |
| Porous materials | 9.9 | 8.2 by Russell, equation (6) |

Table 6 gives a summary of the results. Equations (10) and (11) give better results for solid-solid mixtures and emulsions.

Two conclusions can be drawn:

1. The assumed parabolic distribution of the discontinuous phase for a random distribution of the discontinuous phase in the continuous phase is acceptable.

2. The technique of [1] provides a simple means of calculating thermal conductivity for emulsions, suspensions and porous materials ("air-cell" type) when the portions and properties of the constituents are known.

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TECHNIQUE DE PREVISION DE LA CONDUCTIVITE THERMIQUE DES SUSPENSIONS, DES EMULSIONS ET DES MATERIAUX POREUX

Résumé—Une technique théorique pour prédire la conductivité thermique de mélanges hétérogènes, obtenue précédemment par les auteurs, a été appliquée aux suspensions, aux émulsions et aux matériaux poreux. On montre que la technique basée sur les résultats disponibles dans la littérature, prédit les valeurs de la conductivité thermique à 18,2 pour cent près pour les suspensions, 9,5 pour cent pour les émulsions et 9,9 pour cent les matériaux poreux.

EIN VERFAHREN ZUR VORAUSBESTIMMUNG DER THERMISCHEN LEITFÄHIGKEIT VON SUSPENSIONEN, EMULSIONEN UND PORÖSEN MATERIALIEN.

Zusammenfassung—Ein theoretisches Verfahren zur Vorausbestimmung der thermischen Leitfähigkeit von heterogenen Gemischen, das von den Autoren schon früher entwickelt worden ist, wurde auf Suspensionen, Emulsionen und porösen Materialien angewendet. Durch Vergleich mit Angaben aus der Literatur wird gezeigt, dass die Methode die Werte der thermischen Leitfähigkeit mit einer Genauigkeit von 18 Prozent für Suspensionen, 9,5 Prozent für Emulsionen und 9,9 Prozent für poröse Materialien wiedergibt.

ТЕХНИКА ПРЕДСКАЗАНИЯ ТЕПЛОПЕРЕДАЧИ ЭМУЛЬСИОННЫМИ СУСПЕНЗИЯМИ И ПОРИСТЫМИ МАТЕРИАЛАМИ

Аннотация—Теоретические методы расчета теплопроводности гетерогенных смесей, ранее разработанные авторами, применены к суспензиям, эмульсиям и пористым материалам. На основании данных, имеющих в литературе, показано, что с помощью такой методики можно рассчитать теплопроводность таких суспензий с точностью до 18.2%, эмульсий до 9.5% и пористых материалов до 9.9%.